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THE ATTENUATION OF LONGITUDINAL WAVES IN NON-LINEAR VISCOELASTIC MEDIA†

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Some mechanisms by which longitudinal waves are attenuated in non-linear viscoelastic media, whose equation of state takes into account the presence of structural changes, are demonstrated. It is shown that the degree of attenuation of a longitudinal wave in such media is a linear function of the frequency. © 2002 Elsevier Science Ltd. All rights reserved.

It is usually assumed that when solids are deformed within the limits of elasticity the mechanical characteristics of the material remain unchanged. However, if a medium has a structure, this structure will be broken down on deformation, which leads to a change in the state of the material and, consequently, to a change in its mechanical properties. For soils, this change is much greater than for metals. If the structural changes of solids are taken into account, i.e. if the mechanical characteristics of the material on deformation are assumed to be variable, many facts, observed experimentally [1, 2], can be explained. On the basis of this, an equation of state of soil has been proposed, which takes into account the breakdown in the structure of the material [3]. As an analysis of this equation of state shows, the existence of a relation between the mechanical characteristics of the medium and the degree of breakdown of the structure enables certain non-linear properties of the medium when it is deformed to be explained, even within the framework of the elastic model, while in the framework of the viscoelastic model it has been shown that soils acquire plastic properties when deformed [3].

The non-linearity of the elastic and elastoplastic properties of a medium can also be explained by structural changes (breakdowns) of the material, which lead to variability of its mechanical characteristics [3].

Hence, structural change (breakdown) of materials on deformation is an important factor which must be taken into account in the equations of state, particularly for media such as soil.

1. THE EQUATION OF STATE OF STRUCTURALLY VARIABLE MEDIA

According to the results of experimental research [4, 5], depending on the behaviour of the pressure P when there is a change in the bulk strain θ it is assumed that when there is compression $\theta > 0$ three types of bulk compression diagrams of structurally undamaged soils and rocks are possible (Fig. 1). At the beginning of the deformation process, when the breakdown of the structure of the material is small, the graphs of P against θ are qualitatively the same for all types of compression curves (sections OA in Fig. 1). Along section OA, for many materials the linear Hooke's law is obeyed, but for soils and rocks the curve begins to deviate from linear along this section. Further, along section AC the form of the $P(\theta)$ curves may be different. For curve 1 along section AC the pressure increases as the strain increases. Curve 2 corresponds to the case when the pressure remains constant as the strain increases, while curve 3 corresponds to the case when the pressure falls when the strain increases. Further, after the point C on the compression diagrams, the pressure increases when the strain increases and falls when the strain is reduced.

The different variety of loading diagrams $P(\theta)$ considered were related [3] to breakdowns of the structure of the soil, and it was assumed that when the structure of the material breaks down the bulk (shear) compression modulus changes, and also the values of the other mechanical characteristics; in particular, an interpretation was given of the different forms of compression diagrams of soils.

As a measure of the breakdown of the structure of the soil the parameter $I_s = \theta/\theta_*$ was taken, where θ_* is the value of the bulk strain for which complete structural breakdown of the material occurs ($0 \leq I_s \leq 1$).

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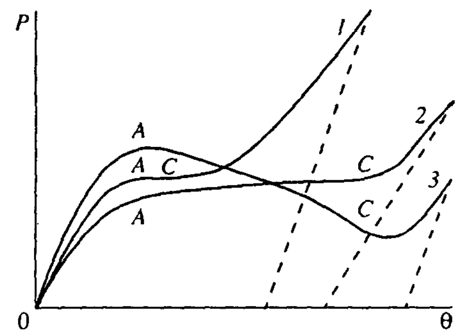

Fig. 1

Breakdown of the structure for a constant load occurs at a different rate depending on the physical and mechanical properties of the material. An analysis of existing experimental data on the compression of soils and rocks [4, 5] shows that the different form of curves 1–3 in Fig. 1 can be explained by the difference in the rates of deformation and breakdown.

On the basis of the results of research on the dynamic breakdown of solids [6] the following relation is taken as the parameter characterizing the breakdown of a material

$$I_R = nV_n/(NV_N) \tag{1.1}$$

where I_R is a dimensionless parameter, n is the number of dislocations and microcracks that occur in the material when it is deformed, V_n is the microvolume of a single dislocation, N is the maximum number of dislocations and microcracks that occur, and V_N is the maximum possible volume of the dislocations in the material.

Following the well-known approach [6], we can assume that N and V_N are constant quantities for a given material. Then, the rate of breakdown is given by the relation

$$dI_R/dt = (NV_N)^{-1}d(nV_n)/dt \tag{1.2}$$

Hence, the rate of breakdown represents the rate of growth of the overall volume of dislocations and microcracks in the material.

When a material is deformed microcracks are produced and their number increases as the rate of deformation increases. At large deformation rates finer fractions (fragments) of the body are formed [5]. It is well known that the greater the strength of the material the longer the process of formation and accumulation of dislocations goes on for different values of the rate of deformation. Judging from existing data on the fracture of rocks [6], the rate of deformation plays an important role in the fracture of materials, and its relation to the rate of breakdown defines the form of the curves on the compression diagram of soils and rocks, shown in Fig. 1.

Curve 1 corresponds to the case when

$$d\theta/dt > \omega dI_R/dt \tag{1.3}$$

where ω is a coefficient of proportionality. When the rate of deformation of the material is much greater than the rate of breakdown of the structure of the material, a "hardening" is observed on the $P(\theta)$ curves. In this case we can assume that the formation and accumulation of dislocations and microcracks has been completed, and agglomeration of the microcracks, their "healing", is occurring, in which case, the values of the density and compression modulus of the material increase.

for curve 2 we can assume

$$d\theta/dt = \omega dI_R/dt \tag{1.4}$$

In this case, a "yield" section is formed on the $P(\theta)$ curves. On this section the process by which dislocations and microcracks are formed and accumulate is accompanied by their partial "healing". The values of the bulk compression modulus of the material decrease slightly.

When a descending section is formed on the $P(\theta)$ diagram, we can assume

$$d\theta/dt < \omega dI_R/dt \tag{1.5}$$

Here the process by which dislocations and microcracks are formed and accumulate is completed by fairly intensive breakdown, and the values of the bulk compression modulus of the material falls sharply.

All the forms of compression diagram shown in Fig. 1 can be observed for the same material depending on the degree of breakdown and the rate of deformation.

The equation of state of a medium, taking into account the breakdown of the structure of the material when it is deformed, based on the linear viscoelastic model for a uniaxial compressed state, has the form [3]

$$\frac{1}{E_D(I_S)}\frac{d\sigma}{dt}+\frac{\mu_0(I_S)}{E_S(I_S)}\sigma=\frac{d\varepsilon}{dt}+\mu_0(I_S)\varepsilon \tag{1.6}$$

where $\sigma = -P$ is the stress and $\varepsilon = \theta$ is the strain.

The values of the dynamic compression modulus function E_D and of the static compression modulus function E_S, and also the viscosity parameter μ_0, are found from the relations

$$\begin{gathered} E_D(I_S)=E_{D^*}\exp(\beta(1-I_S)),\quad E_S(I_S)=E_{S^*}\exp(\alpha(1-I_S)) \\ \mu_0(I_S)=\mu_*\exp(\alpha^0(1-I_S)),\qquad I_S=\varepsilon/\varepsilon_* \end{gathered} \tag{1.7}$$

where E_{D^*}, E_{S^*}, μ_* are the moduli of dynamic and static compression and the volume viscosity parameter of the soil with a completely damaged structure for low rates of deformations, β, α, α^0 are dimensionless factors, characterizing the degree of change of these parameters and ε_* is the value of the strain for which complete breakdown of the structure of the soil occurs.

From relations (1.7) we have that when $\varepsilon = 0$ the initial values of the compression moduli and the viscosity parameter are

$$E_{DN}=E_{D^*}\exp\beta,\quad E_{SN}=E_{S^*}\exp\alpha,\quad \mu_N=\mu_*\exp\alpha^0 \tag{1.8}$$

Hence

$$\begin{gathered} \alpha=\beta+\ln(\gamma_*/\gamma_N),\quad \gamma_*=E_{D^*}/E_{S^*} \\ \gamma_N=E_{DN}/E_{SN},\quad \mu_*=\mu_N/\gamma_*,\quad \alpha^0=\ln\gamma_* \end{gathered} \tag{1.9}$$

The current value $\gamma_I = E_D(I_S)/E_S(I_S)$, which depends on the changes in the structure of the medium, is determined from the relation

$$\gamma_I=\gamma_*\exp(\beta_I(1-I_S)),\quad \beta_I=-\ln(\gamma_*/\gamma_N) \tag{1.10}$$

Hence we obtain $\gamma_I = \gamma_N$ when $I_S = 0$ and $\gamma_I = \gamma_*$ when $I_S = 0$.

However, it is well known [4–6], that γ_* depends very much on the rate of deformation. The relation between the moduli of dynamic and static compression for a given rate of deformation will therefore be taken in the form

$$\gamma=\gamma_I+(\gamma_m-\gamma_I)\left(\frac{d\varepsilon}{\mu_N dt}\right)^{\chi} \tag{1.11}$$

where γ_m is the value of γ for the maximum values of the rate of deformation permissible in the experiments and χ is a dimensionless exponent.

According to expression (1.11), the ratio $E_D(I_S)/E_S(I_S)$ can be considerable for large rates of deformation, difficult to achieve in experiments. The existence of large values of γ has been discussed previously [7].

A medium whose behaviour is determined by Eq. (1.6) possesses properties which give rise to absorption of mechanical energy, i.e. waves in such a medium attenuate. The problem of the propagation of longitudinal waves in a material, whose behaviour is described by Eq. (1.6), is the simplest for investigating the absorbing properties of a medium. The attenuation of longitudinal waves when the values of the parameters of model (1.6) are constant, was investigated previously in [8]. This problem has also been considered [7] for the case of variable viscous properties of the medium.

According to experimental data [9–12], the attenuation factor depends linearly on the frequency of the longitudinal waves over a wide range. The equations of state, which are special cases of Eq. (1.6), do not give a linear relation between the attenuation factor and the frequency [7, 8].

We will consider the problem of the propagation of longitudinal waves in a non-linear viscoelastic medium (1.6).

2. FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

The equations of motion in Lagrangian coordinates for a half-space for the propagation of a plane wave have the form

$$\rho_0 \frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial r} = 0, \quad \frac{\partial u}{\partial r} - \frac{\partial \varepsilon}{\partial t} = 0 \tag{2.1}$$

where u is the velocity of the particles, r is the spatial coordinate and ρ_0 is the initial density of the medium.

The solution of the problem of the propagation and attenuation of longitudinal waves in a half-space is reduced to integration of system (2.1), closed by Eq. (1.6), with boundary conditions in the initial section

$$r = 0: \quad \sigma = 0, \quad t \leqslant 0; \quad \sigma = \sigma_m \sin(\pi t / T), \quad t > 0 \tag{2.2}$$

and on the wave front

$$r = c_0 t: \quad \sigma = -c_0 \rho_0 u = 0, \quad u = -c_0 \varepsilon = 0, \quad c_0 = (E_{DN} / \rho_0)^{1/2}, \quad \mu = \mu_N \tag{2.3}$$

where T is the half-period of the oscillation of the load, c_0 is the propagation velocity of the wave, and σ_m is the maximum stress (the amplitude of the load which produces the wave).

We will use the method of characteristics to solve system (2.1), (1.6). We will assume that, when conditions (2.3) are taken into account, the wave front that arise are weak and, consequently, when they propagate the medium suffers no appreciable breakdowns of the structure. In this case the characteristics of Eqs (2.1), (1.6) remain linear.

The relations along the characteristics have the form

$$\begin{gathered} d\sigma \mp c_0 \rho_0 du = -c_0^2 \rho_0 g(\sigma, \varepsilon) dt, \quad \dot{r} = \pm c_0 \\ d\sigma - c_0^2 \rho_0 d\varepsilon = -c_0^2 \rho_0 g(\sigma, \varepsilon) dt, \quad \dot{r} = 0 \\ g(\sigma, \varepsilon) = \frac{\sigma}{\eta(I_S)} - \mu_0(I_S) \left(\varepsilon - \frac{\sigma}{E_D(I_S)} \right) \\ \eta(I_S) = \frac{E_D(I_S) E_S(I_S)}{(E_D(I_S) - E_S(I_S)) \mu_0(I_S)}, \quad \dot{r} = \frac{dr}{dt} \end{gathered} \tag{2.4}$$

where $\eta(I_s)$ is the bulk viscosity of the medium.

To reduce the number of unknown parameters of the problem, we will introduce the following dimensionless variables

$$\begin{gathered} x = \frac{\mu_N r}{c_0}, \quad t^0 = \mu_N t, \quad \sigma^0 = \frac{\sigma}{\sigma_m}, \quad \varepsilon^0 = \frac{\varepsilon}{\varepsilon_m}, \quad u^0 = \frac{u}{u_m}, \quad u_m = -\frac{\sigma_m}{c_0 \rho_0} \\ f^0 = \frac{f}{\mu_N} = \frac{1}{2\mu_N T}, \quad f = \frac{1}{2T}, \quad \varepsilon_m = \frac{\sigma_m}{E_{DN}}, \quad \mu^0 = \frac{\mu_0(I_S)}{\mu_N}, \quad T^0 = \mu_N T \end{gathered} \tag{2.5}$$

Equation (1.6) and (2.1)–(2.4) then have the form in variables (2.5)

$$\frac{\partial \varepsilon^0}{\partial t^0} + \mu^0 \varepsilon^0 = \frac{\partial \sigma^0}{\partial t^0} + \mu^0 \gamma \sigma^0, \quad \frac{\partial u^0}{\partial t^0} + \frac{\partial \sigma^0}{\partial x} = 0, \quad \frac{\partial u^0}{\partial x} + \frac{\partial \varepsilon^0}{\partial t^0} = 0 \tag{2.6}$$

$$d\sigma^0 \pm du^0 = (\varepsilon^0 - \gamma\sigma^0)\mu^0 dt^0, \quad dx/dt^0 = \pm 1$$
$$d\sigma^0 + d\varepsilon^0 = (\varepsilon^0 - \gamma\sigma^0)\mu^0 dt^0, \quad dx/dt^0 = 0 \tag{2.7}$$

$$x = 0: \quad \sigma^0 = 0, \quad t^0 \leqslant 0; \quad \sigma^0 = \sin(2\pi f^0 t^0), \quad t^0 > 0 \tag{2.8}$$

$$x = t^0: \quad \sigma^0 = \varepsilon^0 = u^0 = 0 \tag{2.9}$$

The solution of the problem was obtained on a computer. Solutions of similar problems were previously obtained by the method of characteristics [7, 8]. Here the value of the bulk viscosity parameter is assumed to be constant; consequently, $\mu^0 = 1$, and the value of γ is found from Eqs (1.10) and (1.11), but it does not take values less than the achievable value in the calculations.

3. RESULTS OF CALCULATIONS AND DISCUSSION

In the problem in question, when longitudinal waves propagate in a medium, the wave front in practice does not bring about any breakdown of the structure, consequently, the current value of γ_1 on the wave front, according to relation (1.10), remains approximately equal to γ_N.

In a numerical solution, to determine the parameters of the wave at the $(i + 1)$th step of the calculations with respect to time, in the case of load (2.8), the rate of deformation will be represented in the form

$$\frac{d\varepsilon}{dt} \cong \frac{\Delta\varepsilon}{\Delta t} = \frac{\varepsilon_i - \varepsilon_{i-1}}{\Delta t} \tag{3.1}$$

When the rate of deformation is determined by relation (3.1), expression (1.11) takes the form

$$\gamma_* = \gamma_N + (\gamma_m - \gamma_N)\left(\frac{\varepsilon_i - \varepsilon_{i-1}}{\mu_N \Delta t}\right)^\chi \tag{3.2}$$

Note that in the calculations, the rate of deformation, after reaching its maximum value, remained constant.

Further, using relation (1.10), we determine the current value γ_1. It should be noted that the assumptions made correspond to the case when the modulus of dynamic compression of the soil $E_D(I_S)$ is constant, while the modulus of static compression $E_S(I_S)$ is variable, i.e. $\gamma_1 = E_D E_S(I_S)$. In this case breakdown of the structure of the soil only affects E_S when I_S changes and, consequently, the value of ε_*. In this case, in Eqs (2.7) the value of $\gamma = \gamma_1$ is variable, but is a known quantity. The characteristics of Eqs (2.1), (1.6) and the relations along the characteristics remain linear.

Hence, we have simplified the numerical solution of the problem and, together with this, the fundamental relations of the proposed improved model of the soil (1.6) remain in force.

The absorption coefficient for the medium (in dimensionless form) can be found from the relation [7–9]

$$\alpha_S^0 = \ln(\sigma_{i-1}^0 / \sigma_i^0)/(x_i - x_{i-1}), \quad \alpha_S c_0 = \alpha_S^0 \mu_N \tag{3.3}$$

where i is the number of sections of the medium, and σ and σ_i^0 are the amplitudes of the stresses in the sections considered.

Computer calculations were carried out for the following values of the dimensionless parameters of the problem: $\gamma_N = 1.1$, $\gamma_m = 10$, $\chi = 0.5$, $\varepsilon_* = 0.2$ and values of the dimensionless frequency of the oscillations of the load (2.2) $f_0 = 5 \times 10^{-4} - 5 \times 10^3$.

For $\mu_N = 10^4\ \mathrm{s}^{-1}$ these values of the frequencies, according to relations (3.1) and (2.5), correspond to the maximum values of the rate of deformation $\varepsilon = 5 \times 10^{-4} - 5 \times 10^{-3}\ \mathrm{s}^{-1}$, and for $\mu_N = 10^6\ \mathrm{s}^{-1}$ the corresponding maximum values are two orders of magnitude less.

For certain versions, the values of χ and ε_* were changed; this will be mentioned later.

As a result of the calculations we obtained the changes in the wave parameters for fixed sections of a soil medium in the form of graphs of the change in the dimensionless stresses σ^0 the strains ε^0 and the velocity of the particles u^0 as a function of the dimensionless variable t^0.

Figure 2 shows the changes with time of the dimensionless stress for fixed sections of a half-space filled with soil for different values of x when $f = 5 \times 10^3\ s^{-1}$. The curve $x = 0$ corresponds to a change in the load acting on the initial section of the half-space and which produces the wave. The amplitude of this load does not change with time (Fig. 2). As the distance from the initial section is increased, after the arrival of the wave the stress increases, reaches a maximum and then changes further also sinusoidally. The maximum value of the stress in these sections is less than in the initial section. The greater the distance from the initial section the less the values of the maximum stress.

Calculations showed that the amplitudes of the first oscillation of the stress in fixed sections of the medium is greater than the amplitudes of subsequent oscillations. However, after five-seven oscillations, as was observed earlier in [7, 8], the amplitude of the oscillations of the stress becomes constant. The difference between the values of the amplitude of the first and steady oscillations of the stress increases as the frequency of the wave increases.

The overall features of the change in the velocity and strains at fixed points of the medium are analogous to the features of the change in the stress, shown in Fig. 2. For different frequencies the overall pattern of the behaviour of the parameters remains the same. Only the amplitudes of the oscillations of the wave parameters vary for fixed points of the medium.

The reduction in the amplitude of the stress with distance for the first and steady oscillations is shown in Fig. 3 for different values of the dimensionless frequency. At low frequencies the attenuation of the wave is small (curves 1 and 2). As the frequency increases the intensity of the wave attenuation increases considerably (curves 3–6). A further increase in the frequency leads to even greater attenuation of the wave with distance.

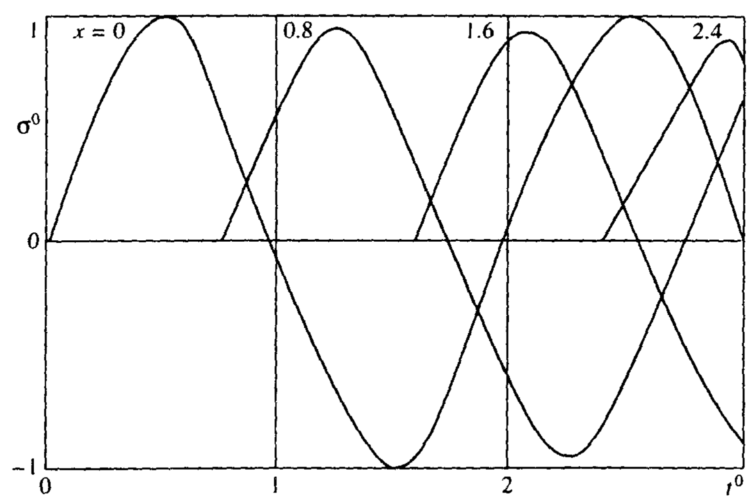

Fig. 2

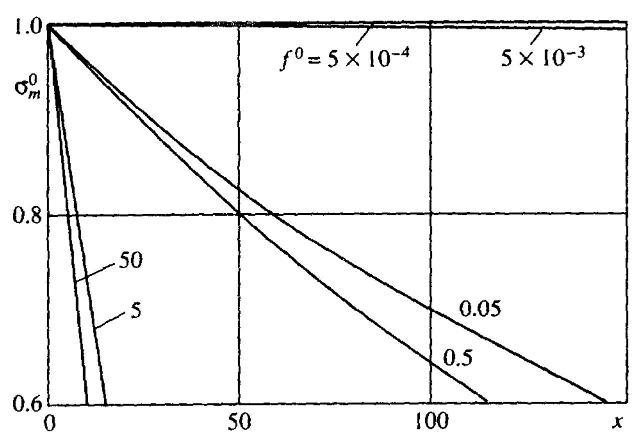

Fig. 3

Using the maximum values (amplitudes) of the stresses, strains and velocity of the particles, from formulae (3.3) we determined the values of the attenuation factor for these wave frequencies. Logarithmic curves of the dimensionless attenuation factor against the frequency of the wave are shown in Fig. 4 by the small circles. The spread in the points is due to the fact that, because of the assumptions made above, the attenuation factors are different in different sections of the medium.

Figure 4(a) corresponds to the case when $\gamma_N = \gamma_* = \gamma_m = 4$, i.e. the models of a standard linear body with constant characteristics. As previously [7,8], in this version of the calculations the attenuation factor remains constant for high-frequency waves. At low frequencies we have a linear relation between the attenuation and the frequency. Beginning at a frequency of $f = 5$ kHz, the attenuation factor does not change. This defect of the standard linear body model is well-known [7–9].

The results of a calculation for the initial values of the parameters of the problem when $\varepsilon_* \to \infty$ ($\varepsilon_* = 10^6$) are presented in Fig. 4(b). An increase in the value of ε_* indicates that the structure of the medium remains practically unchanged during deformation. However, according to expression (3.2), depending on the frequency of the wave, the current values of the ratio of the moduli of the dynamic and static compression γ change. In this case, the attenuation factor at high frequencies increases, but its frequency-dependence remains non-linear. As the frequency of the wave is increased the attenuation factor approaches a constant value.

Figure 4(c) corresponds to a value of $\varepsilon_* = 0.02$, i.e. another limiting case, when the structure of the medium on deformation changes considerably. Here the frequency-dependence of the attenuation factor can be assumed to be approximately linear, though there is a considerable spread in the values of the attenuation factor around the linear relation. At high frequencies the attenuation factor increases.

An increase in the value of ε_* by an order of magnitude ($\varepsilon_* = 0.2$) leads to a reduction in the spread in the values of the attenuation factor around the linear relation (Fig. 4d). In this case the frequency-dependence of the attenuation factor can be assumed to be linear. Note that, when determining the value of the attenuation factor using relation (3.3), from 70 to 100 values of the extrema of the wave in the relations $\sigma^0(t^0)$, $\varepsilon^0(t^0)$ $u^0(t^0)$ and $u^0(t^0)$ were used. Hence, in Fig. 4, for each value of the frequency of the wave we obtained from 70 to 100 values of the attenuation factor. At certain frequencies these values of the attenuation factor are very close and are practically equal, while for other values of the frequencies they have a small spread, which is shown in Fig. 4. In our opinion this is due to the features of the approximate numerical method of solution.

Graphs of the attenuation factor against the frequency, determined using the extremal values of the deformations and the velocity of the particles, are absolutely identical with the results presented in Fig. 4.

Hence, the model of the medium proposed here, which takes into account the structural changes in the medium when deformed and is more sensitive to the rate of deformation, gives a linear relation between the attenuation factor and the frequency of longitudinal waves over a wide frequency band.

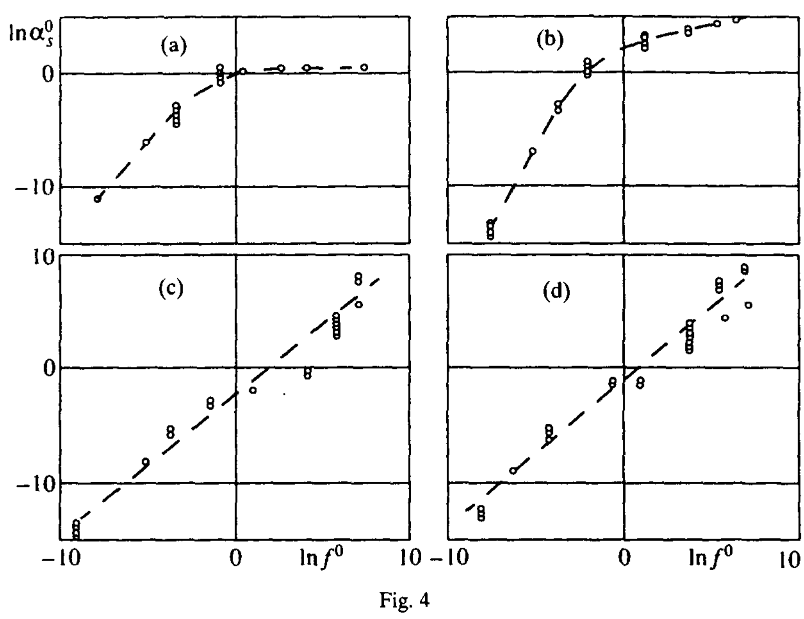

Fig. 4

This confirms the agreement between the proposed equation of state of soil and the results of experiments [9–12]. Hence it follows that the law of soil deformation [3], which takes into account changes in the structure of the material when deformed, gives more accurate results in calculations of the parameters of dynamic processes in soils.

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